

Expression for contact potential

When two regions p & n are joined holes diffuse from p-region to n-region and electrons diffuse from n-region to p-region. The resulting diffusing current cannot build up indefinitely due to creation of opposite electric field at the junction. The electric field E builds up to the point where the net current is zero at equilibrium. The electric field appears in some region about the junction called the transition region and there is an equilibrium potential difference called a contact potential.

At equilibrium, the total electron current is

$$J = ne\mu E + eD \frac{dn}{dx} = 0$$

$$\text{or } -ne\mu E = eD \frac{dn}{dx}$$

$$\text{or } -E = \frac{eD}{ne\mu} \frac{dn}{dx}$$

$$\text{or } \frac{dV}{dx} = \frac{D}{n\mu} \frac{dn}{dx} \quad [E = -\frac{dV}{dx}]$$

From Einstein's relation $\frac{D}{\mu} = \frac{kT}{e}$

$$\therefore \frac{dV}{dx} = \frac{kT}{ne} \frac{dn}{dx}$$

$$dV = \frac{kT}{ne} dn$$

Integrating from $x = -x_p$ to $x = x_n$
we have

$$\int_{-x_p}^{x_n} dV(x) = \frac{kT}{e} \int_{-x_p}^{x_n} \frac{dn}{n}$$

$$\text{or } V(x_n) - V(-x_p) = \frac{kT}{e} [\ln n(x_n) - \ln n(-x_p)]$$

$V_c = V(x_n) - V(-x_p)$ is called built-in potential or barrier potential

$$\therefore V_c = \frac{kT}{e} [\ln n(x_n) - \ln n(-x_p)] \quad \text{--- (1)}$$

If all impurities are ionized then we have $p = N_a$ & $n = N_d$

$\therefore$ The electron concentration in p region is calculated as follows

$$n(-x_p) p(-x_p) = n_i^2$$

$$\therefore n(-x_p) = \frac{n_i^2}{N_a}$$

$$\begin{aligned} \therefore V_c &= \frac{kT}{e} \left[\ln N_d - \ln \frac{n_i^2}{N_a} \right] \\ &= \frac{kT}{e} \ln \left(\frac{N_d N_a}{n_i^2} \right) \quad \text{--- (2)} \end{aligned}$$

which is the voltage necessary to exactly balance the diffusion current under equilibrium condition.

At $x = -x_p$, we have $p_p = N_a \therefore n_p = \frac{n_i^2}{N_a}$

at $x = x_p$ we have $n_n = N_d$, $p_n = \frac{n_i^2}{N_d}$

Thus eqn ② becomes

$$V_c = \frac{kT}{e} \ln \left(\frac{n_n \cdot n_i^2}{n_i^2 \cdot n_p} \right) \quad [n_n = N_d, N_a = \frac{n_i^2}{n_p}]$$

$$= \frac{kT}{e} \ln \left(\frac{n_n}{n_p} \right)$$

$$\therefore \frac{n_n}{n_p} = e^{\left(\frac{eV_c}{kT} \right)} \quad \text{--- ③}$$

ally for holes

$$V_c = \frac{kT}{e} \ln \left(\frac{n_i^2 \cdot p_p}{p_n \cdot n_i^2} \right) = \frac{kT}{e} \ln \left(\frac{p_p}{p_n} \right)$$

$$\therefore \frac{p_p}{p_n} = e^{\left(\frac{eV_c}{kT} \right)}$$

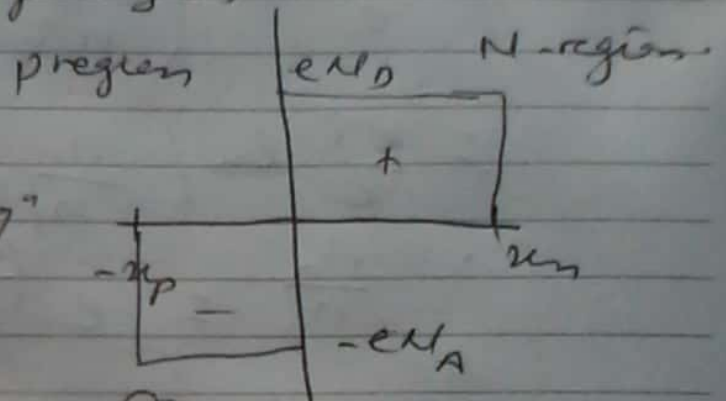
$$\therefore \frac{p_p}{p_n} = \frac{n_n}{n_p} = e^{\frac{eV_c}{kT}}$$

the above relation will be very useful in determination of IV characteristics of the junction.

Width of the depletion layer.

The separation of positive and negative charge densities results in an electric field in a depletion layer.

Let us assume that the space charge region abruptly ends in the n-region at $x = +x_n$ and at $x = -x_p$ in the p-region.



The electric field is found from Poisson's eqⁿ and it is

$$\frac{d^2 V(x)}{dx^2} = -\frac{\rho(x)}{\epsilon} = -\frac{dE(x)}{dx} \quad \text{--- (1)}$$

Where $V(x)$ is the electric potential.

$E(x)$ is electric field, $\rho(x)$ = volume charge density

ϵ = permittivity of the semiconductor

$$\rho(x) = \begin{cases} -eN_A & -x_p < x < 0 \\ +eN_D & 0 < x < x_n \end{cases}$$

Integrating eqⁿ (1) we find electric field in the p-region we have

$$E = \int \frac{\rho(x)}{\epsilon} dx = -\frac{eN_A}{\epsilon} x + C_1$$

The constant of integration C_1 is obtained by setting $E = 0$ at $x = -x_p$. So $C_1 = \frac{eN_A x_p}{\epsilon}$

$$\therefore E = -\frac{eN_A}{\epsilon} (x + x_p) \quad \text{for } -x_p \leq x \leq 0 \quad \text{--- (2)}$$

Eqⁿ (2) is electric field in the p-region.

only the electric field in the n -region

$$E = \int \frac{eN_D}{\epsilon} dx = \frac{eN_D}{\epsilon} x + C_2$$

where $C_2 = -\frac{eN_D x_n}{\epsilon}$ for $E=0$ at $x=x_n$

$$\therefore E = -\frac{eN_D}{\epsilon} (x_n - x) \quad \text{for } 0 \leq x \leq x_n \quad \text{--- (3)}$$

The electric field is continuous at $x=0$. Thus equating eq^s (2) & (3) ~~we get~~ at $x=0$ we get

$$N_A x_p = N_D x_n$$

i.e. no. of -ve charges per unit area in the p region and no. of +ve charges per unit area in the n -region are equal.

Integrating electric field in eqn (3) gives potential in p -region

$$V(x) = - \int E(x) dx = - \int \frac{eN_A}{\epsilon} (x + x_p) dx \\ = \frac{eN_A}{\epsilon} \left(\frac{x^2}{2} + x_p x \right) + C_3$$

Let us set the potential equal to 0 arbitrarily at $x = -x_p$. thus $C_3 = \frac{eN_A}{2\epsilon} x_p^2$

$$\therefore V(x) = \frac{eN_A}{2\epsilon} (x + x_p)^2 \quad \text{--- (4)}$$

It is potential in p -region.

ally The potential in N-region is

$$V_c(x) = \frac{eN_D}{\epsilon} \left(x_n^2 - \frac{x^2}{2} \right) + \frac{eN_A x_p^2}{2\epsilon}$$

The magnitude of potential at $x = x_n$ is equal to the built in potential barrier $V_c(x)$

$$\therefore V_c = \psi(x = x_n) = \frac{e}{2\epsilon} (N_D x_n^2 + N_A x_p^2) \quad \text{--- (5)}$$

we have $x_p = \frac{N_D x_n}{N_A}$

putting x_p in eqn (5) we have

$$x_n = \sqrt{\frac{2\epsilon V_c}{e} \frac{N_A}{N_D} \left(\frac{1}{N_A + N_D} \right)}$$

It is width of space charge in N-region

ally $x_p = \sqrt{\frac{2\epsilon V_c}{e} \frac{N_D}{N_A} \left(\frac{1}{N_A + N_D} \right)}$ is the width of space charge in P-region.

$\therefore$ Total width of depletion is

$$W = x_n + x_p = \left[\frac{2\epsilon V_c}{e} \cdot \frac{N_A + N_D}{N_A N_D} \right]^{1/2}$$

Semiconductor diode

* Equilibrium current across the P-N junction diode

⇒ When P-N junction is formed a potential energy barrier $|e|V_c$ is formed that stops the flow of majority carriers across the junction. This is called dynamic equilibrium. There are electrons in a conduction band of both the N-type and the p-type. The no. of electrons n in the conduction band is given by

$$n = N_c e^{\left(-\frac{E_g - E_F}{k_B T}\right)} \quad \text{--- (1)}$$

The electrons (minority carriers) in the conduction band of the p-region are not impeded by barrier potential from crossing the junction from p-side to N side. The electron current from p to N i.e. $i(p \rightarrow N)$ will be proportional to the total number of electrons in the p-region.

$$\text{i.e. } i(p \rightarrow N) = A \exp\left(-\frac{E_1}{k_B T}\right) \quad \text{--- (2)}$$

Those electrons in N-region having energy equal to or greater than the barrier energy $|e|V_c$ will be able to cross the junction from N side to p-side.

$$\text{i.e. } i(N \rightarrow p) = A N e^{\left(-\frac{E_1 + |e|V_c}{k_B T}\right)} \quad \text{--- (3)}$$

where $f(E \geq |e|V_c)$ is the fraction of these electrons (N_c) with energies equal to or greater than $|e|V_c$. In Si for electrons in conduction band, FD statistics can be approximated by Maxwell-Boltzmann distribution. ~~The fraction~~ the fraction of electrons with energies greater than or equal to given value of E_i is given by Boltzmann factor $\exp(-E_i/k_B T)$.

$$\therefore f(E \geq |e|V_c) = \exp\left(-\frac{|e|V_c}{k_B T}\right) \quad (4)$$

Substituting eqⁿ (1) & eqⁿ (4) in eqⁿ (3) we get

$$i(N \rightarrow P) = A \exp\left(-\frac{E_2 + |e|V_c}{k_B T}\right) \quad (5)$$

Where $E_2 = E_g - E_F$ in the region N.

It is seen that $E_2 + |e|V_c = E_1$ & it follows that

$$i(N \rightarrow P) = i(P \rightarrow N)$$

Thus the flow of electrons from N to P is equal to the flow from P to N. Thus the net current is zero.

④ Flow of charge carriers across the junction under forward and reverse bias.

⇒ The net electron current from N-side to the P-side will be

$$I = i(N \rightarrow P) - i(P \rightarrow N) \quad \text{--- (1)}$$

Here $i(N \rightarrow P)$ represents the flow of electrons (majority carriers) from N $\rightarrow$ P, is proportional to the no. of electrons in the conduction band of the N-region having energies greater than the height of the energy barrier $|e|(V_c - V_0)$

$$\therefore i(N \rightarrow P) = A \exp\left(-\frac{E_2 + |e|(V_c - V_0)}{k_B T}\right) \quad \text{--- (2)}$$

where $E_2 = E_g - E_f$ in the N-side.

and $i(P \rightarrow N)$ represents the flow of electrons (minority carriers) from P to N

$$\text{i.e. } i(P \rightarrow N) = A \exp\left(-\frac{E_1}{k_B T}\right) \quad \text{--- (3)}$$

Thus eqn (1) becomes

$$I = A \exp\left(-\frac{E_2 + |e|(V_c - V_0)}{k_B T}\right) - A \exp\left(-\frac{E_1}{k_B T}\right)$$

Using $E_2 + |e|V_c = E_1$ we get

$$I = A \exp\left(-\frac{E_1 - |e|V_0}{k_B T}\right) - A \exp\left(-\frac{E_1}{k_B T}\right)$$

Daily Notes

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$$i = A \exp\left(-\frac{E_i}{k_B T}\right) \exp\left(+\frac{|e|V_0}{k_B T}\right) - A \exp\left(-\frac{E_i}{k_B T}\right)$$

Letting $i_0 = A \exp\left(-\frac{E_i}{k_B T}\right)$ we get

$$i = i_0 \left[\exp\left(+\frac{|e|V_0}{k_B T}\right) - 1 \right]$$

Here i_0 is due to flow of minority carriers from p to n because E_i is unchanged by the applied voltage. It is called the reverse saturation current.